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LETTER TO THE EDITOR

Conductance of two-dimensional imperfect conductors: does the elastic scattering preclude localization at $T = 0$?

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Abstract. Elastic electron–impurity scattering is proven analytically to prevent interferential localization in two-dimensional wires with more than one conducting channel. An unconventional diffusive regime is found in the length region where the electrons are usually considered as localized. An ohmic, rather than exponential, dependence of the $T = 0$ conductance is predicted, with a length-dependent diffusion coefficient.

Electronic and classical-wave transport in random systems of various dimensionalities has been attracting great attention for scores of years. Numerous attempts in this field have been concentrated around the problem of Anderson localization, whose various aspects have built up to a great extent the understanding of metal–insulator transitions. The prospects for research in this area are substantially determined by the claims of the one-parameter scaling theory of localization [1]. Although universality of the one-parameter scaling has long been challenged [2, 3], attempts were made (and still are made) to improve the scaling approach in view of its relative convenience and simplicity [4, 5]. They were stimulated considerably by experimental findings of unexpectedly anomalous transport in dilute two-dimensional (2D) electron and hole systems [6]. Also, unconventional experimental results have stimulated the development of different approaches to the problem of quantum transport in disordered 2D systems (see, e.g., the discussion in reference [7]); the most intriguing expectations are associated with the Coulomb interaction of carriers [8, 9]. Yet the transport theories with e–e interaction still cannot claim to have general acceptance, because of the substantial controversy as regards interpreting the role of the interaction within different domains of parameters corresponding to diffusive [10] and localized [11, 12] regimes.

In spite of an ample variety of theoretical approaches to the problem of localization, some points in this field are still vague, and therefore attract intensive research. The important obstacle is insufficient mathematical grounds for localization in 2D and 3D random systems, contrary to the 1D case [13], where rigorous results have been obtained for arbitrary strength of disorder. Meanwhile, it is instructive to point out that the elaboration of practical asymptotic methods for calculating the disorder-averaged many-particle characteristics (conductivity, density–density correlation, etc), leaving aside the detailed spectral analysis, was even more important for the theory of 1D random systems than development of the mathematical foundation [14–17]. Similar arguments in favour of localization, or against it, have not yet been found for 2D and 3D systems, except as regards some aspects of the weak-localization problem [18, 19].

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The objective of this letter is to derive a quantum theory, i.e. based on waveguide ideology, well adapted for the analysis of charge transfer in weakly disordered 2D conductors. This theory can be constructed with the same rigour as that in references [14–17], due to the problem of quantum transport in 2D *waveguide systems* being reduced exactly to a set of purely one-dimensional subsidiary problems. Although substantial complication of the potentials arises as a requirement for such a reduction (the problems turn out to be non-Hermitian), the dynamic properties of one-dimensional systems can now be subjected to canonical analysis beyond the scaling hypothesis, RMT, etc.

We consider a two-dimensional rectangular sample of length L in the x-direction and width D in the y-direction, where non-interacting electrons subject to a static random potential are confined between the hard-wall lateral boundaries $y = \pm D/2$, while in the direction of the current (x) we suppose the system open. The dimensionless conductance $g(L)$ (in units of $e^2/\pi\hbar$) is computed from the linear response theory [20], whence at zero temperature the formula follows:

$$g(L) = -\frac{4}{L^2}\int\int_L \mathrm{d}x\ \mathrm{d}x' \sum_{n,n'=1}^{\infty} \frac{\partial G_{nn'}(x,x')}{\partial x}\frac{\partial G^*_{nn'}(x,x')}{\partial x'}. \tag{1}$$

Here $G_{nn'}(x,x')$ is the retarded one-electron Green function in the coordinate-mode representation, i.e. Fourier transformed over the transverse coordinate. This function obeys the equation

$$\left[\frac{\partial^2}{\partial x^2} + k_n^2 + \mathrm{i}0 - V_n(x)\right]G_{nn'}(x,x') - \sum_{\substack{m=1\\(m\neq n)}}^{\infty} U_{nm}(x)G_{mn'}(x,x') = \delta_{nn'}\delta(x-x') \tag{2}$$

where $k_n^2 = k_F^2 - (n\pi/D)^2$ is the longitudinal mode energy, k_F is the Fermi wavenumber, and $U_{nm}(x)$ is the mode matrix element of the 'bulk' random potential $V(\boldsymbol{r})$ which is assumed to have zero mean and the binary correlator

$$\left\langle V(\boldsymbol{r})V(\boldsymbol{r}')\right\rangle = \mathcal{Q}W(\boldsymbol{r}-\boldsymbol{r}') \tag{3}$$

where $\boldsymbol{r} = (x,y)$. The angular brackets in equation (3) stand for impurity averaging; the function $W(\boldsymbol{r})$ is normalized to unity and has the correlation radius r_c.

From the technical point of view it is important that the diagonal matrix element $V_n(x) \equiv U_{nn}(x)$ is initially separated in equation (2) from off-diagonal elements, so that the matrix $\|U_{nm}\|$ governs inter-mode transitions only. This enables us to strictly reduce the problem of finding the entire set of functions $G_{nn'}(x,x')$ to the solution of a subset of purely one-dimensional closed equations for the diagonal mode functions $G_{nn}(x,x')$. The exact 'one-dimensionalization' procedure is sketched out below.

First, we introduce the auxiliary (trial) Green function $G_n^{(V)}(x,x')$ obeying the equation

$$\left[\frac{\partial^2}{\partial x^2} + k_n^2 + \mathrm{i}0 - V_n(x)\right]G_n^{(V)}(x,x') = \delta(x-x') \tag{4}$$

and Sommerfeld's radiative conditions [21] at the strip ends $x = \pm L/2$, which seem natural for an open system. Then, turning from equation (2) to the consequent integral equation

$$G_{nn'}(x,x') = G_n^{(V)}(x,x')\delta_{nn'} + \sum_{m=1}^{\infty}\int_L \mathrm{d}t\ \mathsf{R}_{nm}(x,t)G_{mn'}(t,x') \tag{5}$$

with the kernel

$$\mathsf{R}_{nm}(x,t) = G_n^{(V)}(x,t)U_{nm}(t) \tag{6}$$

one can express all of the off-diagonal mode elements G_{mn} via the diagonal ones G_{nn} by means of the linear operator $\hat{\mathsf{K}}$:

$$G_{mn}(x, x') = \int_L \mathrm{d}t\ \mathsf{K}_{mn}(x, t) G_{nn}(t, x'). \tag{7}$$

The equation for the matrix elements $\mathsf{K}_{mn}(x, x')$ of $\hat{\mathsf{K}}$ results directly from equation (2):

$$\mathsf{K}_{mn}(x, x') = \mathsf{R}_{mn}(x, x') + \sum_{\substack{k=1 \\ (k \neq n)}}^{\infty} \int_L \mathrm{d}t\ \mathsf{R}_{mk}(x, t) \mathsf{K}_{kn}(t, x'). \tag{8}$$

This equation belongs to a class of multi-channel Lippmann–Schwinger equations that are known to be extremely singular in general, in contrast to their single-channel counterparts [22]. However, by choosing the trial Green function $G_n^{(V)}$ as a zero approximation for $G_{nn'}$ and perturbing it by the inter-mode potentials $U_{nm}(x)$ only, we manage to avoid the above-mentioned singularity. Therefore the solution of equation (8) can be written in the form

$$\hat{\mathsf{K}} = (\mathbf{I} - \hat{\mathsf{R}})^{-1} \hat{\mathsf{R}} \boldsymbol{P}_n \tag{9}$$

where $\hat{\mathsf{R}}$ is an operator acting in the mixed coordinate-mode space (x, n) and specified by the matrix elements (6). It is important that the space indicated contains all of the waveguide modes except the nth mode itself. The projection operator $\boldsymbol{P}_n$ makes the mode index of any operator that stands next to $\boldsymbol{P}_n$ (either on the left or on the right) equal to n.

From equations (2), (7), (9) we obtain the exact closed one-dimensional equation for each diagonal function $G_{nn}(x, x')$ separately:

$$\left[\frac{\partial^2}{\partial x^2} + \kappa_n^2 + \mathrm{i}0 - V_n(x) - \Delta\hat{\mathcal{T}}_n \right] G_{nn}(x, x') = \delta(x - x') \tag{10}$$

with $\kappa_n^2 = k_n^2 - \langle \hat{\mathcal{T}}_n \rangle$ and $\Delta\hat{\mathcal{T}}_n = \hat{\mathcal{T}}_n - \langle \hat{\mathcal{T}}_n \rangle$. The operator $\Delta\hat{\mathcal{T}}_n$ acts on the variable x only, since from equation (9) it follows that the operator $\hat{\mathcal{T}}_n$ is a two-dimensional T-matrix [22] enveloped by the projective operators $\boldsymbol{P}_n$:

$$\hat{\mathcal{T}}_n = \boldsymbol{P}_n \hat{\mathcal{U}} (\mathbf{I} - \hat{\mathsf{R}})^{-1} \hat{\mathsf{R}} \boldsymbol{P}_n = \boldsymbol{P}_n \hat{\mathcal{U}} (\mathbf{I} - \hat{\mathsf{R}})^{-1} \boldsymbol{P}_n. \tag{11}$$

$\hat{\mathcal{U}}$ is the inter-mode scattering operator in (x, n) space, specified by matrix elements

$$\langle x, k | \hat{\mathcal{U}} | x', m \rangle = U_{km}(x) \delta(x - x').$$

Hereinafter, when analysing equation (10), we regard a set of the renormalized energies κ_n^2 $(n = 1, 2, \ldots)$ as representing the new 'unperturbed spectrum' of the system, instead of the original spectrum $\{k_n^2\}$. The perturbation theory will now be developed making use of the appropriate zero-mean potentials $V_n(x)$ and $\Delta\hat{\mathcal{T}}_n$.

To complete the one-dimensionalization we express the conductance (1) through the diagonal Green functions G_{nn} and the trial functions $G_n^{(V)}$ (both one-dimensional!) In this letter we focus on the case of *weak electron–impurity scattering* specified by the inequalities

$$k_F^{-1}, r_c \ll \ell \tag{12}$$

with $\ell = 2k_F/\mathcal{Q}$ denoting a *semiclassical* mean free path evaluated for a δ-correlated 2D random potential, i.e. $W(\boldsymbol{r}) = \delta(\boldsymbol{r})$ in equation (3). The conditions (12) allow us to expand the operator $\hat{\mathsf{K}}$, equation (9), to lowest order in the inter-mode operator $\hat{\mathsf{R}}$, which in turn enables us to replace the exact operator $\hat{\mathcal{T}}_n$ from equation (11) by its approximate value:

$$\hat{\mathcal{T}}_n \approx \boldsymbol{P}_n \hat{\mathcal{U}} \hat{G}^{(V)} \hat{\mathcal{U}} \boldsymbol{P}_n \tag{13}$$

with the operator $\hat{G}^{(V)}$ defined by the matrix elements

$$\langle x, k | \hat{G}^{(V)} | x', m \rangle = \delta_{km} G_m^{(V)}(x, x').$$

Applying now equations (7), (9), and (13) to equation (1), we arrive at the following expression for the impurity-averaged conductance:

$$\langle g(L)\rangle = -\frac{4}{L^2}\sum_{n=1}^{\infty}\int\int_L \mathrm{d}x\ \mathrm{d}x'\left[\left\langle \frac{\partial G_{nn}(x,x')}{\partial x}\frac{\partial G^*_{nn}(x,x')}{\partial x'}\right\rangle \right.$$
$$+\frac{\mathcal{Q}}{D}\sum_{\substack{m=1\\(m\neq n)}}^{\infty}\int_L \mathrm{d}y\left\langle G_m^{(V)*}(x,y)\frac{\partial}{\partial x}G_m^{(V)}(x,y)\right\rangle$$
$$\left.\times\left\langle G_{nn}(y,x')\frac{\partial}{\partial x'}G_{nn}{}^*(y,x')\right\rangle\right]. \tag{14}$$

At this point it is useful to discuss the spectral properties of the quantum-mechanical system governed by equation (10). First, the term $\langle\hat{\mathcal{T}}_n\rangle = \Delta k_n^2 - \mathrm{i}/\tau_n^{(\varphi)}$ which modifies the initial spectrum $\{k_n^2\}$ can be readily calculated. In the limit (12), an explicit form of the function $W(\boldsymbol{r})$ is not so important, and we obtain from equation (13)

$$\Delta k_n^2 = \frac{\mathcal{Q}}{D}\sum_{\substack{m=1\\(m\neq n)}}^{\infty}\mathcal{P}\int_{-\infty}^{\infty}\frac{\mathrm{d}q}{2\pi}\,\frac{\widetilde{W}(q+k_n)}{k_m^2-q^2} \tag{15a}$$

$$\frac{1}{\tau_n^{(\varphi)}} = \frac{\mathcal{Q}}{D}\sum_{\substack{m=1\\(m\neq n)}}^{N_c}\frac{1}{4k_m}\left[\widetilde{W}(k_n-k_m)+\widetilde{W}(k_n+k_m)\right]. \tag{15b}$$

In equation (15*a*), the symbol $\mathcal{P}$ stands for principal value, and $\widetilde{W}(q)$ is the Fourier transform of the correlation function $W(\boldsymbol{r}-\boldsymbol{r}')$ over y, y', and $x-x'$. The summation in equation (15*b*) is restricted by the number $N_c = [k_F D/\pi]$ of *conducting channels* (extended waveguide modes), because only for $n \leqslant N_c$ is the disorder-averaged diagonal-in-n Green matrix $\langle\hat{G}^{(V)}\rangle$ in equation (13) essentially complex. The purely real addendum (15*a*) to the original mode energy k_n^2 is small under the conditions of equation (12), so it can be omitted. At the same time, the 'level broadening' $1/\tau_n^{(\varphi)}$, which can be interpreted as the inverse phase-breaking time for the nth-mode state, relates to the mean level spacing as $N_c/k_F\ell$, and thus cannot be omitted in the framework of the weak-scattering approximation in general. Just the quantity (15*b*) is of crucial importance for the further analysis.

We emphasize that the level broadening (15*b*) implies the presence of other extended modes with $m \neq n$ in the conductor. For extremely narrow strips with $N_c = 1$, the imaginary term is not present in the renormalized mode spectrum, as the sum (15*b*) contains no terms in this case. Thus the system should exhibit true one-dimensional properties. Specifically, the electrons can be transferred within two regimes only, *ballistic* and *localized*, and the conductance of such a wire is decreased *exponentially* with the length L exceeding the *localization length* $\xi_1 = 4L_b \sim \ell$, the quadruple Born backscattering length [23].

On increasing the conductor width, as soon as the wire ceases to be single mode ($N_c \geqslant 2$), the situation changes drastically. The nth-mode spectrum is modified jointly by both the potentials $V_n(x)$ and $\Delta\hat{\mathcal{T}}_n$, and acquires the level broadening (15*b*). We thus come to the necessity of analysing the condition of (one-dimensional!) localization in *lossy* media, though *no inelastic scattering* was initially involved in the problem. The appropriate comprehensive theory is beyond the scope of a letter, and will be given in a more extensive publication [24].

Here we emphasize that in studying spectral properties of a system governed by equation (10), one should clearly distinguish between the *direct* intra-mode scattering due to the *local* potential $V_n(x)$ and *indirect* intra-mode scattering due to the *operator* potential $\hat{\mathcal{T}}_n$. The intra-mode potential $V_n(x)$ gives rise to the coherent localization effect, just as in the case

with $N_c = 1$; see reference [23]. This potential causes local (in x) *elastic* $n \to n$ transitions, so the effect is purely interferential. Meanwhile, from equations (11), (13) it follows that the potential $\hat{\mathcal{T}}_n$ can also be associated with the $n \to n$ scattering, but via all of the other modes, i.e. excluding the nth mode. Pictorially, this can be thought of as diffusion in the mode space with return to the initial mode.

It is justified thus to regard the operator potential $\hat{\mathcal{T}}_n$ just as governing the *inter-mode* scattering within the effectively 'single-mode' problem (10). This scattering can lead both to the coherent localization (due to the potential $\Delta\hat{\mathcal{T}}_n$) and to the uncertainty of a mode state due to the term $\mathrm{i}/\tau_n^{(\varphi)}$ arising from the strong complexity of the trial functions $G_m^{(V)}(x, x')$ at $m \leqslant N_c$. This duality of the inter-mode scattering, especially the appearance of the 'phase-breaking' term $\mathrm{i}/\tau_n^{(\varphi)}$, in spite of the scattering due to the potential $\hat{\mathcal{T}}_n$ being effectively *intra-mode* (i.e., at first sight elastic), has a clear physical explanation. It certainly results from the *probabilistic* nature of electron transitions through intermediate mode states $m \neq n$ (intrinsic to the potential $\hat{\mathcal{T}}_n$) with mode energies *different* from k_n^2. This *hidden inelasticity* is precisely the reason for the strong complexification of the quasi-particle spectrum (15).

In the final stage we discuss the role of the inter-mode scattering over the whole range of the conductor length by estimating the Born scattering rate $1/\tau_n^{(\mathcal{T})}$ which determines the fundamental frequency of the states presumably localized by the 1D random potential $\Delta\hat{\mathcal{T}}_n$. Estimation of the operator norm $||\Delta\hat{\mathcal{T}}_n||^2$ with the use of equation (13) yields

$$\frac{\tau_n^{(\varphi)}}{\tau_n^{(\mathcal{T})}} \sim \frac{1}{\cos^2\vartheta_n}\min\left(1, \frac{L/D}{k_F\ell}\right) \tag{16}$$

where ϑ_n is a 'sliding angle' of the mode n with respect to the x-axis, $|\sin\vartheta_n| = n\pi/k_F D$. The level broadening for an nth mode exceeds the level spacing provided that the wire is not extremely stretched along the x-axis, i.e. if the length L does not fall into the interval

$$L \gg Dk_F\ell \sim N_c\ell. \tag{17}$$

Yet even within this interval the level spacing $1/\tau_n^{(\mathcal{T})}$ due to the potential $\Delta\hat{\mathcal{T}}_n$ *cannot exceed* the level broadening $1/\tau_n^{(\varphi)}$. Consequently it is useless to seek the traditional interferential localization at any length of the multi-mode ($N_c \geqslant 2$) conductor.

To illustrate the above statement, we find the average conductance (14) for different lengths in the relatively easy case $N_c \gg 1$. The exact mode function G_{nn} can be obtained from the equation

$$G_{nn}(x, x') = G_{nn}^{(0)}(x, x') + \left(\hat{G}_{nn}^{(0)}\,\Delta\hat{\mathcal{T}}_n\,\hat{G}_{nn}\right)(x, x') \tag{18}$$

which stems directly from equation (10), where the 'unperturbed' function $G_{nn}^{(0)}(x, x')$ obeys equation (10) with $\Delta\hat{\mathcal{T}}_n = 0$. With the estimate (16) taken into account, one can solve equation (18) perturbatively in $\Delta\hat{\mathcal{T}}_n$. In doing so, the addition to the conductance emerges; this is similar to the second term $\langle g^{(2)}(L)\rangle$ on the r.h.s. of equation (14), but proportional to a higher degree of the small interaction strength $\mathcal{Q}$. The potential $\Delta\hat{\mathcal{T}}_n$ can thus be removed from equation (10) and the inter-mode scattering taken into account through the dephasing rate $1/\tau_n^{(\varphi)}$ and the term $\langle g^{(2)}(L)\rangle$. The potential $V_n(x)$, though different from $\Delta\hat{\mathcal{T}}_n$ in its physical meaning, can also be removed from equation (10) because of the relative smallness of its norm, $\langle\|\hat{V}_n\|^2\rangle/\langle\|\Delta\hat{\mathcal{T}}_n\|^2\rangle \sim N_c^{-1}$. Then the Green function G_{nn} can be replaced in equation (14) by its 'unperturbed' expression (the main approximation in $N_c^{-1} \ll 1$)

$$G_{nn}^{(0)}(x, x') = \frac{1}{2\mathrm{i}k_n}\exp\left\{\left[\mathrm{i}k_n - 1/(\ell\cos\vartheta_n)\right]|x - x'|\right\} \tag{19}$$

which nonetheless includes most of the inter-mode scattering effects.

As to the functions $G_m^{(V)}(x, y)$ in equation (14), at $L \ll N_c\ell$ we can put the potential $V_m(x) \equiv 0$ since the mth-mode localization length found from equation (4) with the use of the method of reference [23] is

$$\xi_m = \frac{16\pi}{3} N_c \ell \cos^2 \vartheta_m / \widetilde{W}(2k_m) \sim N_c\ell.$$

In this case the second term in equation (14) turns out to be $-1/8$ of the first one, i.e. not parametrically small. Yet in the limit (17), all of the functions $G_m^{(V)}(x, y)$ are localized, and therefore the second term in equation (14) is negligibly small.

On the basis of the above arguments, we arrive at the following asymptotic expressions for the conductance (14), disregarding weak-localization corrections governed by the intra-mode potentials $V_n(x)$:

$$\begin{aligned} &\text{(i) } L < \ell: && \langle g(L)\rangle \approx N_c \\ &\text{(ii) } \ell \ll L \ll N_c\ell: && \langle g(L)\rangle \approx \frac{7\pi}{32} N_c\ell/L \gg 1 \\ &\text{(iii) } N_c\ell \ll L: && \langle g(L)\rangle \approx \frac{\pi}{4} N_c\ell/L \ll 1. \end{aligned} \tag{20}$$

The result given in equation (20) allows us to distinguish three regimes of charge transport in multi-mode conductors, none of them localized in the anticipated sense. Regime (i) corresponds to entirely ballistic transport, both from the semiclassical and quantum standpoints. In regimes (ii) and (iii) the semiclassical motion should be regarded as diffusive. The difference between them is that in regime (ii) all of the mode states could be considered extended in the absence of the inter-mode scattering, whereas in regime (iii) they would all be localized due to the potentials $V_n(x)$. In both diffusive regimes (ii) and (iii), the conductance exhibits purely ohmic (inversely proportional to L) behaviour, but with different (classical) diffusion coefficients. Note that just in regime (iii), where all of the *trial states* would be localized if the inter-mode scattering was disregarded, the result given by the classical kinetic theory is exactly reproduced. No exponential decay of the conductance appears at any length and width of the system provided that $N_c \geqslant 2$.

To conclude, the $T = 0$ conductance of a 2D finite-size disordered metal strip was calculated. The interferential localization was shown to manifest itself strongly only for single-mode, i.e. purely 1D, conductors. In commonly examined square-shaped multi-mode samples, the electron transport is diffusive as long as $L \gg \ell$, the semiclassical mean free path. For any extended (propagating) mode in a multi-mode strip, all of the other extended modes can be thought of as an effective phase-breaking reservoir destroying quantum interference and hence the exponential localization.

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